

Formal Languages: Final Exam

Your completed exam is due at 10am on Monday. You may use any sources, provided you cite them fully and make clear what is your own work and what is not. At least one of the questions is pretty tough—remember that you do not need to get everything exactly right to get a good grade. Some of the questions are a little imprecise about what exactly is required. We want you to demonstrate your understanding of the material: the more impressive your answer, the more credit you'll get. Good luck.

1. For each of the following languages,

- decide where it belongs in the Chomsky Hierarchy,
- give an argument that convinces us you're right,
- describe a machine that recognizes it (if possible),
- describe a grammar that produces it (if possible).

a) The language with symbols $(,), [$ and $]$ whose valid strings are those with correctly matching and nested parentheses and brackets.

b) The Fibonacci numbers, in some suitable representation.

c) The subset of $\{0, 1\}^*$ whose strings have even length and no more than 3 contiguous 1's.

d) The set $\{a^m b^n c^{m+n} : m, n \geq 0\}$.

2. For one of the languages in Question 1 implement the machine and test at least 10 strings. Also implement the grammar and generate at least 10 strings.

You can use programming languages and libraries as you like, but be clear about where you got it and how it works. The more you do yourself (as opposed to loading library X to do it all), the more brownie points you get. We're not asking for a general purpose machine, but one for this specific language.

3. Show P is closed under concatenation and complement. Show that NP is closed under union and concatenation.

4. EITHER

Let ϕ be a formula in 3CNF form. A \neq -assignment to the variables of ϕ is one in which each clause has a pair of literals with unequal truth values. That is, in a satisfying assignment each clause has at least one false literal (one way to think of it is that each clause is like an XOR but with three inputs—the clause evaluates to “true” when one or two of the literals is true and evaluates to “false” otherwise.)

Let \neq -SAT be the set of 3CNF-formulas that have a \neq -assignment. Show that \neq -SAT is NP-complete.

(If you want some hints consult Sipser (2nd ed.) problem 7.24)

OR

Jitt Mahollis, faculty member at Winston College, decides to spend the summer break living in the woods around campus. He wants to limit the weight of his backpack and is having trouble choosing which items to take. Jitt assigns to each potential item i a weight $w(i)$ and a value $v(i)$ —his problem is to determine the maximum value the items that come in under some maximum weight M . Having just taught a course on the theory of computation, Jitt decides to formulate the problem as a language problem BACKPACK, which expresses the question of whether a given value can be achieved within a given weight.

Define such a language BACKPACK and show that the resulting problem is NP-complete.

5. Let P be the language $\{\langle M \rangle : M \text{ is a TM that accepts all palindromes}\}$. Show that P is undecidable. (Recall that a *palindrome* is a string that reads the same forwards and backwards. Hint: it is possible to adapt problem 7.28 of Sipser to give a solution.)