$$\boxed{\int_{-\infty}^{+\infty} e^{ikx}\,dx = 2\pi\,\delta(k)}$$ to a physicist, anyway.

Well, literally it doesn't converge. But... let's define it as

$$\lim_{\varepsilon\to 0}\left\{\int_{-\infty}^{0} e^{i(k-i\varepsilon)x}\,dx + \int_{0}^{\infty} e^{i(k+i\varepsilon)x}\,dx\right\}$$

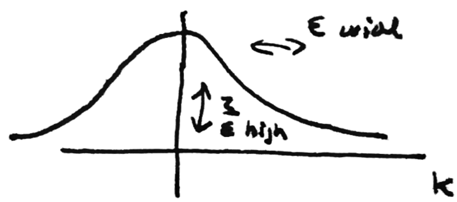
$$\left.\frac{e^{+\varepsilon x+\dots}}{ik+\varepsilon}\right|_{-\infty=x}^{0=x} \qquad \left.\frac{e^{-\varepsilon x+\dots}}{ik-\varepsilon}\right|_{0=x}^{\infty=x}$$

This is basically a cheat in complex-integration...

$$\lim_{\varepsilon\to 0}\left\{\frac{1}{ik+\varepsilon} - \frac{1}{ik-\varepsilon}\right\} = \lim_{\varepsilon\to 0}\left\{\frac{-ik+\varepsilon}{k^2+\varepsilon^2} - \frac{-ik-\varepsilon}{k^2+\varepsilon^2}\right\}$$

Let's define

$$\boxed{f_\varepsilon(k) \equiv \frac{2\varepsilon}{\varepsilon^2+k^2}}$$



as $\varepsilon\to 0$

for $k \ll \varepsilon$, $f_\varepsilon(k) \approx \frac{2}{\varepsilon}$

for $k \gg \varepsilon$, $f_\varepsilon(k) \approx \frac{2\varepsilon}{k^2}$

What is the area under it?

$$\int_{-\infty}^{+\infty}\left(\frac{2\varepsilon}{\varepsilon^2+k^2}\right)dk = \cancel{\int_{-\infty}^{+\infty} \frac{2\,(\varepsilon/k)}{\dots}} = \int_{-\infty}^{+\infty} \frac{2}{1+(k/\varepsilon)^2}\cdot d(k/\varepsilon)$$

let $z = k/\varepsilon = \tan\theta = \frac{\sin\theta}{\cos\theta}$ and $k: -\infty \to +\infty$ is $-\frac{\pi}{2} \to +\frac{\pi}{2}$ for θ

$dz = d(k/\varepsilon) = d\theta\left(\frac{\sin\theta}{\cos^2\theta}\cdot\sin\theta + \frac{\cos\theta^2}{\cos^2\theta}\right) = \left[\frac{d\theta}{\cos^2\theta}\right]$

$1+\tan^2\theta = \left[\frac{1}{\cos^2\theta}\right]$

independent of ε!

so

we get $$\int_{-\infty}^{+\infty} f_\varepsilon(k)\,dk = 2\int_{-\pi/2}^{+\pi/2}\frac{d\theta/\cos^2\theta}{1/\cos^2\theta} = 2\pi$$

$$\Rightarrow \lim_{\varepsilon\to 0} f_\varepsilon(k) = 2\pi\,\delta(k)$$