Addition of velocities in special relativity

The lorent transform is

$$\boxed{\begin{aligned} ct' &= \gamma\,(ct' - \beta x) \\ x' &= \gamma\,(x - \beta ct) \end{aligned}}$$

$\beta = \dfrac{v}{c}$ $\quad \gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

Frame x' moves at v with respet to x.

or in vector & matrix notation

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

Now say we have another frame x'' moving at v' relative to x'

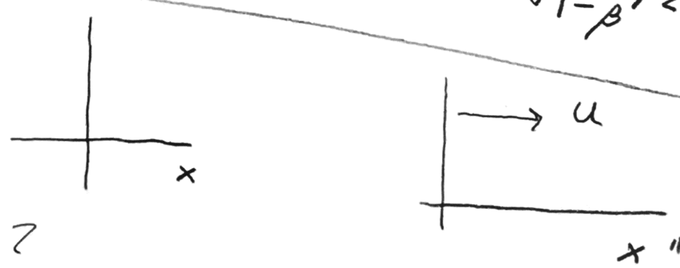
$$\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \gamma' & -\beta'\gamma' \\ -\beta'\gamma' & \gamma \end{bmatrix} \begin{bmatrix} ct' \\ x' \end{bmatrix}$$

$\beta' = \dfrac{v'}{c}$

$\gamma' = \dfrac{1}{\sqrt{1-\beta'^2}}$

The transform from x to x'' is some velocity u.

Is that a Lorentz Transform? And if so, what is u in terms of v and v'?



Answer: yes. Define $\beta_u = u/c$

$\gamma_u = \dfrac{1}{\sqrt{1-u^2/c^2}}$

$$\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \gamma' & -\beta'\gamma' \\ -\beta'\gamma' & \gamma' \end{bmatrix} \begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

$$= \begin{bmatrix} \gamma\gamma' + \beta\beta'\gamma\gamma' , & -\beta\gamma'\gamma - \beta'\gamma\gamma' \\ -\beta'\gamma\gamma' - \beta\gamma\gamma' , & \beta'\beta\gamma\gamma' + \gamma\gamma' \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} \gamma_u & -\beta_u\gamma_u \\ -\beta_u\gamma_u & \gamma_u \end{bmatrix}$$

or

$$\boxed{\begin{aligned} \gamma\gamma'(1+\beta\beta') &\stackrel{?}{=} \gamma_u \\ \gamma\gamma'(\beta+\beta') &\stackrel{?}{=} \beta_u\gamma_u \end{aligned}}$$

Divide $\Rightarrow$ $\boxed{\beta_u = \dfrac{\beta+\beta'}{1+\beta\beta'}}$

addition of velocities!

Claim: the rest works too.