Exponential growth

Jim Mahoney | April 7 2020

In [1]:
```
from matplotlib import pyplot as plt
import numpy as np
```

Simulations start with differential equations.

Here a classic describing the rate of growth of some variable $x(t)$ - say, the number of people infected with a virus - by saying that the rate of change dx/dt (i.e. the number of new people who get infected per day) is proportional to to the variable itself (i.e. the number of people who already have it) times some constant a.

$$\frac{dx}{dt} = ax$$

Of course there are many more complications in the real world, but let's see where this simple model goes.

difference equations

Rather than work with a continuous function $x(t)$ we will instead use a list of numbers, each a constant time step dt apart.

```
[ x(t0), x(t0 + dt), x(t0 + 2*dt), ...]
```

which is just a list of numbers $x[i]$ where i is the time index meaning "at time $t_0 + i * dt$.

Then the differential equation becomes a difference equation :

$$\frac{x_{i+1} - x_i}{dt} = a * x_i$$

or

```
x[i+1] = a * dt * x[i] + x[i] = x[i] * (1 + a * dt)
```

We can pick some sample values for the starting value x_0, the constant a and step size dt, loop over this, and then plot it.

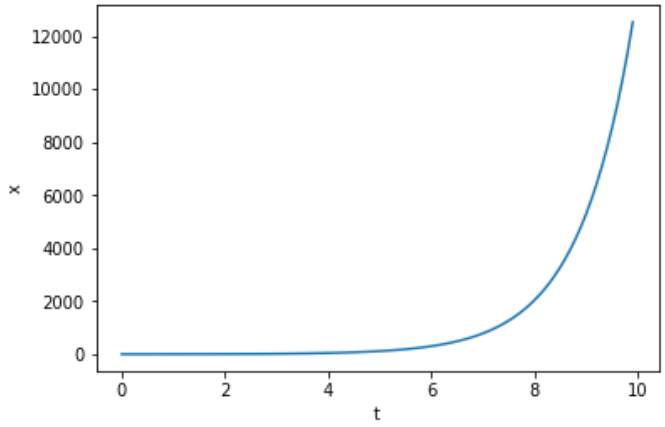
In [9]:
```
x0 = 1
a = 1
dt = 0.1

n = 100    # number of points

x = [0.0 for i in range(n)]
t = [dt * i for i in range(n)]

x[0] = x0
for i in range(n-1):
    x[i+1] = (1 + a * dt) * x[i]

plt.plot(t, x)
plt.xlabel('t')
plt.ylabel('x')
plt.show()
```



This is exponential growth ... and this is what's going on now with covid-19.

In [ ]:
```
```