Permutation

Question

There are four blood types A, B, AB and O. Blood can also be Rh^+ and Rh^-. Finally, a blood donor can be male or female. How many different ways can a donor have his or her blood labelled?

Question

Suppose an electronics store offers a three component stereo system for \$250. A buyer must choose :-

- one amplifier,
- one tuner, and
- one pair of speakers.

If the store has two models of amplifiers, three models of tuners, and two speaker models, how many different stereo systems could a consumer purchase?

Number of amplifiers × number of tunes × number of pairs of speakers = number of systems

2 × 3 × 2 = 12

Blood type : 4 possibilities

Rh factor : 2 possibilities

Gender : 2 possibilities

$4 \times 2 \times 2 = 16.$

Fundamental Counting Principle

Let $T_1, T_2, T_3, \ldots, T_n$ be a sequence of n conditions. Suppose that T_1 can occur in m_1 ways,

T_2 can occur in m_2 ways,

.

and so on until finally T_n can occur in m_n ways.

Then the number of ways of satisfying the conditions $T_1, T_2, T_3, \ldots, T_n$ in succession is given by the product.

$$m_1 \cdot m_2 \cdot m_3 \cdots m_n$$

Question

The letters A, B, C, D, and E are to be used in a four-letter ID card. How many different cards are possible if repetitions are permitted?

Question

A fair coin is tossed 10 times. How many different sequences of Heads and Tails are possible?

Question

Using the letters A, B, C, D and E, how many different ID cards are possible if repetitions of letters are not permitted?

Soln

ID card

$$5 \cdot 5 \cdot 5 \cdot 5 = 5^4$$
$$= 625.$$

Note there are four spaces to fill, each with 5 letters

Coin - toss

There are 10 spaces to fill, each with two options, heads or tails.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{10}$$
$$= 1024$$

ID Card no repetition

$$5 \cdot 4 \cdot 3 \cdot 2 = 120$$

Definition

A permutation is an arrangement of distinct objects in a definite order.

e.g. abc and bca are two possible permutations of three elements a, b, c.

Factorial

For any natural number n

$$n! = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$$

$n!$ is read as "n factorial".

$0!$ is defined as 1.

e.g.

$0! = 1$

$1! = 1$

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Question

1. Find $8!$

2. Evaluate each

(a) $\dfrac{9!}{4!}$

(b) $\dfrac{12!}{10!}$

3. Consider a race with 10 runners. In how many different ways can the runners finish first, second and third (assuming no ties)?

Solution

1. $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 40320$

2. (a) $\frac{9!}{4!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 15120$

(b) $\frac{12!}{10!} = \frac{12 \cdot 11 \cdot \cancel{10!}}{\cancel{10!}} = 132$

3. Any one of 10 runners can finish first : 10

9 remaining runners can finish second : 9

8 remaining runners can finish third - 8

$$10 \cdot 9 \cdot 8 = 720$$

$$\frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} = 720$$

Permutation Rule

The arrangement of n objects in a specific order using r objects at a time is called a permutation of n objects taking r objects at a time. It is written as $_nP_r$ (or alternatively $P(n,r)$) and

the formula is

$$_nP_r = P(n,r)$$

$$= \frac{n!}{(n-r)!}$$

Example

1. In how many ways can a president, vice president, secretary, and treasurer be selected from a committee of fifteen people?

2. Six people attend a movie and all sit in the same row with six seats.

a) Find the number of ways the group can sit together

b) Find the number of ways that the group can sit together if two people in the group must sit side-by-side.

c) Find the number of ways the groups can sit together if two people in the group refuse to sit side-by-side.

Solution

1. $n=15$, $r=4$. $\quad {}_{15}P_4 = \dfrac{15!}{(15-4)!} = \dfrac{15!}{11!}$

$$= \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot \cancel{11!}}{\cancel{11!}} = \underline{\underline{32760}}$$

2. a) $n=6$, $r=6$. $\quad {}_6P_6 = \dfrac{6!}{(6-6)!} = \dfrac{6!}{0!} = \dfrac{6!}{1} = \underline{\underline{720}}$

b) Think of the two people who must sit together as a single object and count the number of arrangements of five objects (AB), C, D, E, F.

So $n=5$ and $r=5$. $\quad {}_5P_5 = \dfrac{5!}{(5-5)!} = \dfrac{5!}{0!} = \dfrac{5!}{1} = 120$

There are 120 arrangements with A and B reversed (BA), C, D, E, F.
So total is $120 + 120 = \underline{240}$

c). From a), there are 720 possible seating arrangements.
- From b), there are 240 arrangements with two specific people sitting next to each other.

Thus there are $720 - 240 = \underline{480}$ arrangements of where two specific people are not seated together.