Notes

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Expected Values

Let a probability experiment $X = \{X_1, X_2, \ldots, X_n\}$ take n distinct values. We let $P(X_1)$ be the probability of outcome X_1, let $P(X_2)$ be the probability of outcome X_2, etc until we let $P(X_n)$ be the probability of outcome X_n.

Expected value

What happens in a probability situation over the long run.

e.g. Person flips a coin many times. They can expect that "heads" will occur about half the time.

The *expected value* E (or *expectation*) is given by the following formula:

$$E = X_1 \cdot P(X_1) + X_2 \cdot P(X_2) + \cdots + X_n \cdot P(X_n).$$

For example, a die is rolled and the expected outcome is given by

$$E = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$$

, which gives that

$$E = \frac{21}{6}.$$

In other words, $E = 3.5$.

Loss and Gain

We can use the idea of expected values of a probability experiment to measure the loss or gain that you would experience by participating in a probability experiment.

For example, one thousand tickets are sold for a mp3 player valued at $400. What is the expected value of the gain if a person purchases one ticket?

	WIN	LOSE
Gain, X	$400	-$1
Probability, $P(X)$	$\frac{1}{1000}$	$\frac{999}{1000}$

The expected value is given by the following:

$$E = \$400 \cdot \frac{1}{1000} + (-\$1) \cdot \frac{999}{1000}$$

, and so

$$E = -\$0.5999$$

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