Exercises — Functions and Coordinate Systems

1. The area of a circle as a function of its radius is $A(r) = \pi r^2$.

   (a) A verse of the Bible reads "And he made a molten sea, ten cubits from the one brim to the other: it was round all about, and his height was five cubits: and a line of thirty cubits did compass it about." (I Kings 7, 23; II Chronicles 4, 2.) What is the value of π according to this verse?

   (b) In the Rhind Papyrys (ca. 1650 BC) the area of a circle is calculated as follows: "Example of a round field of a diameter 9 khet. What is its area? Take away $\frac{1}{9}$ of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. Therefore it contains 64 setat of land." What is the value of π according to the Rhind Papyrys?

   (c) $\star$ Show that $\pi > 3$ by inscribing a regular hexagon in a circle.

2. The enemy has one cannon movable along a shoreline $y = -\frac{1}{4}$ and one cannon located at a fortress off the shore at the point $\left(0, \frac{1}{4}\right)$. You are required to sail between them. What path should you choose to minimize the danger of being hit?

3. (a) What are the coordinates of the point (x, y) reflected in the x-axis?

   (b) What are the coordinates of the point (x, y) reflected in the line $y = x$?

4. (a) $\star$ Show that a line through the point $(-1, 0)$ with rational slope cuts the unit circle $x^2 + y^2 = 1$ in a point (x, y) with rational coordinates. ("Rational" means "ratio of two integers.")

   (b) $\star$ The formulas you found for x and y can be used to generate Pythagorean triples, i.e. to find numbers a, b, c such that $a^2 + b^2 = c^2$. How?

5. $\star$ "It is clear, in fact, that if the points whose coordinates are rational were alone regarded as real, the incircle of square and the diagonal of the square would not intersect, since the coordinates of the point of intersection are irrational." (Poincaré, *Science and Hypothesis*, p. 26.) Explain.

6. Conic sections.

   (a) In three-dimensional x, y, z-space, the equation $x^2 + y^2 = z^2$ represents a cone. Why?

   (b) $\star$ This imples that "conic sections" are curves of degree two. Consider for example the plane $y = 1$. Why is $y = 1$ a plane? What will be its intersection with the cone? What type of conic section is it?

   (c) What are the possible shapes of the shadow cast by a lampshade with a circular rim?

   (d) "Gnomon" is a fancy word for a stick standing in the ground. The tip of its shadow traces a curve as the sun moves. What type of curve is it? How can you find north by using the stick and the curve?