Exercises — Primes and divisibility

1. Find the prime factorizations of the following integers.
   (a) 48.
   (b) 120.
   (c) 39.
2. The prime factorization of 9945 is $3^2 \cdot 5 \cdot 13 \cdot 17$.
   (a) Is 9945 divisible by 26? Explain.
   (b) Is 9945 divisible by 51? Explain.
3. If the sum of three different primes is even, what is the smallest of the three?
4. McNuggets can be bought in quantities of 6, 9 or 20.
   (a) It is impossible to buy precisely 43 McNuggets. Why?
   (b) But we can buy precisely 44, 45, 46, 47, 48 or 49 McNuggets. How?
   (c) Therefore it is possible to buy any given number of McNuggets greater than 43. Why?
   (d) If we are allowed to sell McNuggets back in quantities of 6, 9 or 20, then we can arrange to end up with any given number of McNuggets. How? Hint: find a scheme of buying and selling that leaves us with just one McNugget, then repeat.
5. Prove that $\sqrt{3}$ is irrational.