A Whirlwind Tour Of Mathematics

Credits: 4
Level: Introductory
Location: Sci216, MTh 1.30–2.50pm, Fall 2007
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Website: `http://cs.marlboro.edu/courses/fall2007/math_tour/home`

1 Blurb

Do you want a thorough understanding of the most important and deep theorems in every branch of mathematics? Do you want to achieve this in a four credit course from a standing start? Good luck with that—you won't manage it in this course. Instead, we'll look at six to ten topics, chosen for their accessibility and beauty, and drawn from a broad range of subdisciplines of math. Some potential topics are listed in the next section and on the website.

No prior mathematical experience is expected

2 Topics and Schedule

Below is a list of topics, along with approximate time to be spent on each of them. You'll notice that there is more time allocated than we have in the semester. Rather than working into January, we'll pick which topics we like the the sound of as a group and pursue those. Brief descriptions of the potential topics will appear on the website as I write them. More topics might show up too. If there is something you'd especially like to see, lobby me to include it.

Topology of Surfaces (1–2 weeks)	Irrational Numbers (1 week)
Cryptography (1 week)	Fermat's Last Theorem (2 weeks)
Latin Squares (1–2 weeks)	Imaginary Numbers (1 week)
Symmetry (1 week)	Coloring Maps (1 week)
The Infinitesimal (1 week)	Game Theory (1–2 week)
The Platonic Solids (1 week)	The Infinite (1–2 weeks)
Chance (1–2 weeks)	Chaos and Fractals (1–2 weeks)
P vs. NP (2 weeks)	What is Math? (1–2 weeks)

I've spoken to several friends in the world of math about this course and my choice of topics. They are united in two views. First, there is at least one vitally important topic

that I left off the list. Second, there is at least one topic on the list unworthy of inclusion. Where they disagree entirely with each other is on what else should be included and what should be left off.

If you're worried that this seems disorganised, hold tight, it's about to get worse. Every now and again, a math story makes front page news. Recent examples include Perelman's proof of the Poincaré Conjecture that earned him a $1,000,000 prize (or would have done if he'd been bothered enough to collect it) and the mapping of E8, a complicated mathematical structure of interest to physicists (links to the stories available on the website). If such a story shows up during the semester, we'll drop whatever we're doing and investigate the math behind the story.

So what *is* fixed at the moment? After our initial discussion on Monday we'll start with Topology of Surfaces and move onto Irrational Numbers the following week. In the topology segment we'll be asking what it means for something to be a surface, and meet some familiar examples, such as the sphere and the torus (donut) and some more exotic ones, such as the Möbius strip and the Klein bottle. Irrational Numbers will be the first of several segments in which we look at instances in history where the conventional mathematics of the day could not deal with new problems that arose, and what was done to resolve these issues.

At the end of the course we'll consider the question "What is math?" along with related ones such as "Is math discovered or invented?" and "What do mathematicians mean when they describe a theorem as elegant or beautiful?". This is where everything will come together (I hope). A few of the topics will have obvious common elements. Beyond that there should be deeper themes. Despite seeming to flit between disparate topics, everything we will have looked at is grouped under the heading of Math. Why? What makes it math? Why should we care?

Finally, the last session(s) will be devoted to your presentations.

3 Assignments and Grades

Each topic will have at least one reading and at least one assignment. The reading will usually happen before or early on in the topic; the assignment will usually be due a week after we've finished dealing with that topic in class. Here's how the grades are assigned.

Short version: The more you engage with the material, the better your grade will be and the more rewarding you'll find the course.

Less short version: There will be four components to the grade—graded assigments, progress assignments (including attendance and class participation), a "mathematical con-

versation" and a final project. The two types of assignment are worth a total of 60% of your grade. The exact ratio of graded to progress assignments will depend on the topics we choose. The conversation is worth 10%. Think of the conversation as a mild version of an oral exam on a topic of your choice. It will happen around Week 6 or 7. The final project is worth 30%. This will require a short (10–15 min) presentation of a mathematical idea that we have not discussed in class and an accompanying paper.

The least short version is on the website, where more details of the assignments, conversation and projects are available.

4 What to do now (Assignment 1)

The first class will meet on Monday. Before then I'd like you to think about encounters with math in your life up until this point. Interpret this as broadly as you like. Then, write a few paragraphs (about one side, two at the most) on an incident that has some mathematical component that has affected you. The course will start with a group discussion on your thoughts on this topic.

After that we'll be gluing bits of paper together and then cutting them up (a.k.a. topology).

5 A few final comments

You are expected to be aware of the college's policy on academic integrity and to abide by it. It can be found on the college website, and is linked from the course website. Please come and talk to me if anything is unclear.

This is a four credit course. This means you should be expecting to spend in the region of twelve hours each week (including class time) working on it.

Lastly, be aware that there will be some difficult ideas to sink your teeth into. I'll do my best to cut them into bitesize pieces, but you'll have to do your own chewing. You will sometimes feel bewildered or frustrated. Bewilderment and frustration are natural states for the mathematician. However, you'll find that the increased insight, and the buzz of success, that comes when you crack a tricky problem is worth the effort.