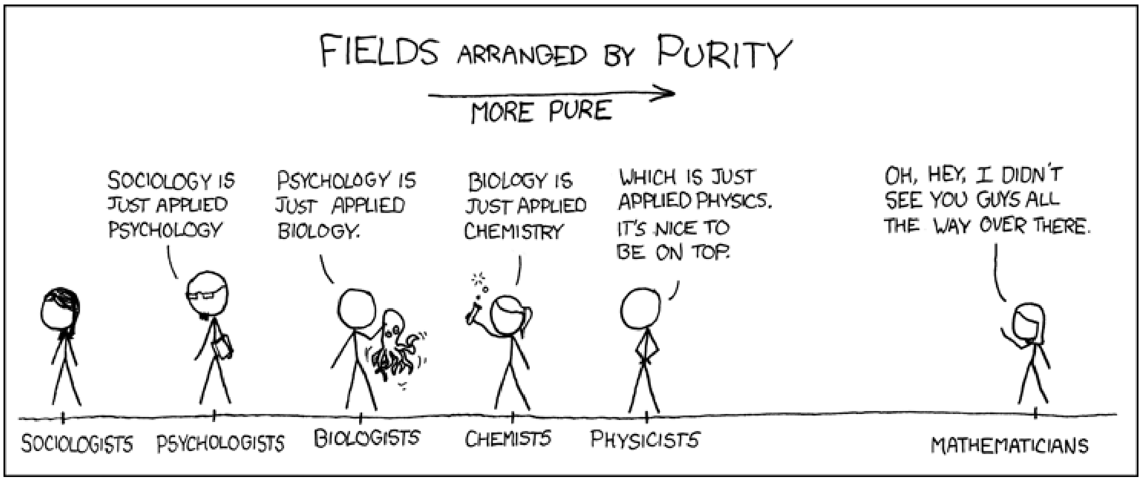
Real Analysis

Credits: 4
Level: Advanced
Prerequisite: Calculus or permission of instructor
Location and Times: Sci217, MWF 9.30–10.20am
Instructor: Matt Ollis, matt@marlboro.edu
Website: http://cs.marlboro.edu/courses/fall2008/analysis/home



A comic strip from www.xkcd.com

1 Introduction

Mathematics is a vibrant field that has two-way connections with many other disciplines, so how did the stereotype in the above cartoon come about? Well, courses like this one for a start.

We'll be looking inward at the structure of mathematical objects and arguments with the aim of putting results about the real numbers and functions of real numbers on a sound theoretical footing. Most of the theorems we'll prove are ones you have seen and used in a calculus class. However, our emphasis in this course will be on why those results are true rather than what we can use them for.

Real Analysis, being the gateway into one of the three major branches of math (algebra and geometry are the other two), is a topic every math student must study. The skills of deductive and careful reasoning also stand students of other disciplines in good stead, and for this reason it is commonly considered to be a desirable course for prospective students to have taken by graduate schools in many scientific (both natural and social) fields.

2 Content

The following topics will form the core of the course:

- **Foundations.** The necessary logic, proof techniques and set theory. Most likely this will be your first exposure to the "axiomatic approach" that characterises modern mathematics.

- **Real Numbers.** A descriptive (and axiomatic) approach to the real numbers. What *is* that number line thingy that you thought you'd understood for over a decade?
- **Convergence and Continuity.** Sequences, limits and continuous functions. All familiar from calculus, but here we'll go much more deeply into their workings.
- **Differentiation.** Again you'll recognise the big picture from calculus, but I hope by this point in the course you'll be eager to revisit the topics with your newly acquired mathematical perspective.

This core corresponds, with some additions and omissions to be announced as we go, to the first six chapters of Lay's *Analysis with an Introduction to Proof* [1]. This is the course text; you will need access to a copy.

We'll start with a refresher of what you know from calculus and, more importantly, look at what your calculus course glossed over in order to press on to the important applicable results. This will take about a week and we'll need two weeks or so for each of the above four bullets. This leaves us some time to either follow interesting tangents or press on into further topics.

Interesting tangents could include a more considered look at the foundations of math (logic and set theory), a constructive approach to the real numbers (we'd build them out of the integers, via the rationals), more topological results, and more pathological fractal curves. The two most natural further topics are integration (including the fundamental theorem of calculus) and power series.

3 Grading

Your grade is determined as follows: 40% final exam; 30% homework assignments; 20% quizzes; 10% in-class proofs. There will be three quizzes; your best two each contribute 10% to the grade. At least three times each, and probably more, I'll assign you an individual proof to work on and present in class. Your best three will together contribute the 10%.

Attendance, class participation, and prompt submission of homework are expected. Your performance in these areas will influence your final result by up to one letter grade.

4 Academic Integrity

You are expected to be aware of the college's policy on academic integrity and to abide by it. It can be found on the college website, and is linked from the course website. Please come and talk to me if anything is unclear.

References

[1] Steven R. Lay, *Analysis with an Introduction to Proof (4th ed.)*, Pearson Prentice Hall, New Jersey (2005).