Calculus 1 **Midterm Review** **Fall 2013**

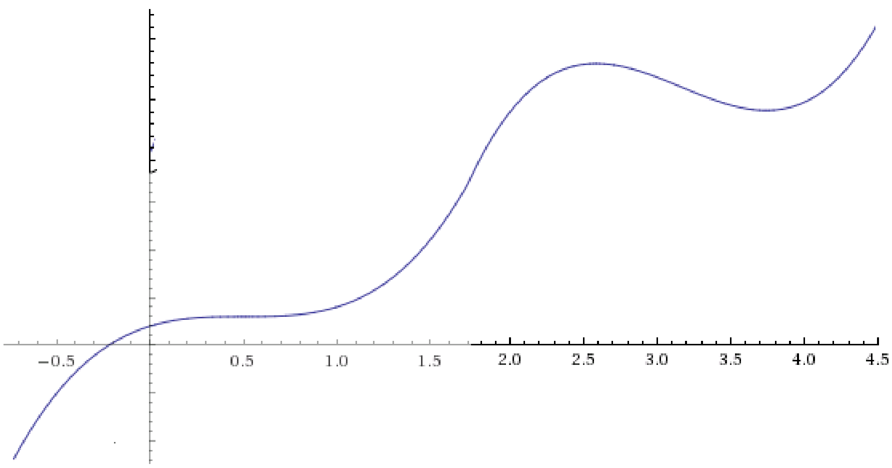
These questions are similar to the questions that will be on the midterm. The topics appearing here are not necessarily all that may appear on the midterm (anything in your notes and homework is also fair game).

1. If $f(x) = 3x^2$, $g(x) = e^x$ and $h(x) = 2 + x$, find $f(g(h(-2)))$.

2. State the domain of the function $f(x) = \dfrac{\sqrt{x^2 - 9}}{x}$.

3. Write out the definition of the derivative of $f(x) = \sqrt{x^2 + 9}$. Do not evaluate the limit.

4. Sketch the derivative of the following function



5. Find y'. Do not simplify.

   (a) $y = e^e - \pi$
   (b) $y = e^{\sin(\theta)}$
   (c) $xy^4 + x^2 y = x + 3y$
   (d) $y = \cos(x)^x$
   (e) $y = \tan^2(\sin(\theta))$
   (f) $y = \cos(e^{\sqrt{\tan(3x)}})$
   (g) $y = \ln(x^2 e^x)$
   (h) $y = \csc(x - \sec x)$
   (i) $y = \sin^2 x \dfrac{\sqrt[3]{3x - 2}}{(e^x - x^2)^{22}}$

6. Find the equation of the tangent line to the curve $y = \sin x \cos x$ at the point $(\frac{\pi}{4}, \frac{1}{2})$.

7. For the following problem, you may leave your answers in terms of exponents and logarithms. (You do not need to approximate with a calculator.)

   A bacteria culture contains 200 cells initially and grows at a rate proportional to its size. After half an hour the population is 360 cells.

(a) Find an equation for the number of bacteria after t hours.

(b) Find the number of bacteria after 4 hours.

(c) Find the rate of growth of the bacteria after 4 hours.

(d) When will the population reach $10,000$?

8. Winnie the Pooh lost his balloon. It is rising into the air at a constant speed of 5 feet per second. Christopher Robin is on a bike, cycling along a straight road at a speed of 15 feet per second. When he passes under the balloon, it is 45 feet above him. How fast is the distance between Christopher Robin and the balloon increasing 3 seconds later?

9. On a cold winter day in Marlboro, you build a snowman whose head is shaped like a large sphere. In a burst of creativity, you name the snowman Frosty. The next day, there's a sudden temperature shift and Frosty's head starts melting, although its spherical shape remains the same. Frosty's head is melting so fast that the surface area is decreasing at a rate of 1 square inch per minute. Find the rate at which the diameter of Frosty's head is decreasing when his head is 1 foot (12 inches) in diameter. (Hint. You'll need some or all of the following information: given a sphere of radius r, the volume of the sphere is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$.) Bonus: How fast is the volume of Frosty's head decreasing at the same time?