Calculus 1 **Assignment 6** **Fall 2013**

1. Show that for any real number c, the graph of $y = x^4 + 4x + c$ has at most two x-intercepts.

2. A number a is a fixed point of a function f if $f(a) = a$. Show that if $f'(x) \neq 1$ for all real numbers, then f has at most one fixed point.

3. Sketch the graph of one function that satisfies all of the given conditions:

   $f'(3) = f'(-2) = 0$,

   $f'(x) < 0$ if $-2 < x < 3$,

   $f'(x) > 0$ if $3 < x < 5$,

   $f(x) = -2$ if $x > 5$ or $x < -2$,

   $f''(x) < 0$ if $-2 < x < 0$,

   $f(x)$ has an inflection point at $(0, -1)$, and

   $f(x)$ is continuous everywhere.

4. Graph the function $f(x) = x^5 - 40x$. Label any maxima, minima, inflection points, and intercepts.

5. Hermione is designing a poster advocating the liberation of house elves. She needs the poster to contain 750 square inches of printing and have margins of 2 inches at the top and bottom and 1 inch at the sides. What dimensions should she have the printed area of the poster be in order to minimize the amount of posterboard needed?