Definitions and Theorems

For each of these, you should be able to state the definition, and quickly come up with a couple of examples.

Linear systems

1. Linear system
2. Reduced row echelon form
3. Elementary row operations
4. Free variable
5. Particular solution vs. homogeneous solution to a system of linear equations
6. Theorem: The general solution to a system of linear equations is a "particular + homogeneous".

Matrices

1. Diagonal matrix
2. Square matrix
3. Identity matrix
4. Transpose of a matrix (and properties)
5. Symmetric matrix
6. Product of two matrices
7. Trace of a matrix
8. Block matrix
9. Invertible (or nonsingular) matrix (and properties)
10. How to compute the inverse of an invertible matrix
11. How to use the inverse of a matrix to solve a system of equations.
12. Theorem: A square matrix M is invertible if and only if the homogeneous system $MX = 0$ has no nonzero solutions.
13. Elementary matrices (corresponding to the elementary row operations)
14. Determinant of a matrix in terms of permutations, and how to calculate using the minors of a matrix.

15. Determinant of the inverse of a matrix.
16. Theorem: M is invertible if and only if $\det M \neq 0$.

Vector spaces and linear transformations

1. Vector space
2. Subspace of a vector space (and the theorem that to check if a subset is a subspace, it is enough to check multiplicative and additive closure)
3. Span of a set of vectors
4. Linear combination of a set of vectors
5. Linearly (in)dependent vectors
6. Basis of a vector space
7. Dimension of a vector space
8. Linear transformation
9. Examples of linear transformations: horizontal and vertical shears, scalings, rotations of the plane
10. Matrix of a linear transformation
11. Null space and range of a linear transformation
12. Rank-Nullity Theorem: Given a linear transformation $L : V \to W$, where V is finite-dimensional, we have
$$\dim(V) = \dim(\text{null space}(V)) + \dim(\text{range}(V)).$$

Eigenvectors, eigenvalues, and diagonalization

1. Eigenvalue, eigenvector of a linear transformation/matrix
2. Eigenspace corresponding to a given eigenvector
3. Characteristic polynomial of a matrix
4. Characteristic equation of a matrix (set the characteristic polynomial equal to zero)
5. Coordinates of a vector $v \in V$ relative to a basis $\mathcal{B} = \{v_1, \dots, v_n\}$ of V
6. Change-of-basis matrix from a basis $\mathcal{B}$ of a vector space V to the standard basis of V.
7. Diagonalizable matrix
8. Similar matrices
9. How to diagonalize a matrix using the change-of-basis matrix from a basis of eigenvectors to the standard basis of a vector space (especially $\mathbb{R}^n$)

Inner product spaces and Gram-Schmidt Orthonormalization

1. Dot product of two vectors in $\mathbb{R}^n$ in terms of their coordinates, and in terms of their norms and the angle between them
2. Norm (magnitude) of a vector in $\mathbb{R}^n$
3. Fact: two vectors in $\mathbb{R}^n$ are perpendicular (orthogonal) if and only if their dot product is zero.
4. Inner product space
5. Length (norm) of a vector in an inner product space
6. Cauchy-Schwartz and Triangle inequalities
7. Orthogonal set in an inner product space
8. Orthonormal set in an inner product space
9. Orthonormal basis of an inner product space
10. Orthogonal projection of a vector onto another vector in an inner product space
11. Gram-Schmidt orthonormalization of a basis
12. Orthogonal complement of a subspace
13. Direct sum of two subspaces of a vector space
14. Theorem: Given a subspace W of an inner product space V, the space V decomposes as $V = W \oplus W^{\perp}$.
15. Theorem: The eigenvalues of a real symmetric matrix are real.
16. Theorem: The eigenvectors corresponding to distinct eigenvalues of a real symmetric matrix are orthogonal.
17. Orthogonal matrix
18. Orthonormal matrix
19. Theorem: A real symmetric matrix M is diagonalizable and the change-of-basis matrix P from M to its diagonalization D is an orthogonal matrix.

Practice Problems

All taken from DeFranza-Gagliardi

- pp. 89-90: exercises 1, 9, 10
- (optional) pp. 90-91: odd-numbered problems between 1 and 25
- pp. 123-124: 3, 5, 8, 9
- (optional) pp. 125-126: odd-numbered problems between 1 and 33
- pp. 194-195: 2, 4, 6 (for part f, instead find the transition matrices from B to the standard basis and from T to the standard basis. How could you use these to find the transition matrix from B to T? From T to B?), 8
- (optional) pp. 195-197: odd-numbered problems between 1 and 25
- pp. 270-272: 1, 4, 7a-d, 8a-d
- (optional) pp. 272-274: odd-numbered problems between 1 and 31
- pp. 316-318: 2, 4, 6, 8
- (optional) pp. 318-319: odd-numbered problems between 1 and 29
- pp. 404-405: 2a-f, 4, 10
- (optional) pp. 406-407: odd-numbered problems between 1 and 15, odd-numbered problems between 25 and 35, and also 24 ,32, 36,