

NSC 562
DISCRETE MATHEMATICS
FALL 2017

HOMEWORK 2

- (1) (Exercise 15 from *Concrete Mathematics* Chapter 4) Euclid numbers are defined by the recurrence

$$e_0 = 2$$
$$e_n = e_0 \cdot e_1 \cdots e_{n-1} + 1 \text{ for } n \geq 1.$$

Does every prime occur as a factor of some Euclid number e_n ? Explain.

The rest of the problems come from Daniel J. Velleman's *How to Prove It*.

- (2) (Exercise 2 from *Introduction*) Make some conjectures about the values of n for which $3^n - 1$ is prime or the values of n for which $3^n - 2^n$ is prime. (You might start by making a table similar to what we did in class, Figure 1 in the book.)
- (3) (Exercise 4 from *Introduction*) Find five consecutive integers that are not prime.
- (4) (Exercise 3 from §1.1) Analyze the logical forms of the following statements:
- (a) Alice and Bob are not both in the room.
 - (b) Alice and Bob are both not in the room.
 - (c) Either Alice or Bob is not in the room.
 - (d) Neither Alice nor Bob is in the room.
- (5) (Exercise 6 from §1.1) Let S stand for the statement “Steve is happy” and G for the statement “George is happy.” What English sentences are represented by the following expressions?
- (a) $(S \vee G) \wedge (\neg S \wedge \neg G)$.
 - (b) $[S \vee (G \wedge \neg S)] \wedge \neg G$.
 - (c) $S \vee [G \wedge (\neg S \vee \neg G)]$.
- (6) (Exercise 2 from §1.2) Make truth tables for the following formulas:
- (a) $\neg[P \wedge (Q \vee \neg P)]$.
 - (b) $(P \vee Q) \wedge (\neg P \vee R)$.
- (7) (Exercise 8 from §1.2) Use truth tables to determine which of the following formulas are equivalent to each other:
- (a) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$.
 - (b) $\neg P \wedge Q$.
 - (c) $(P \vee \neg Q) \wedge (Q \vee \neg P)$.
 - (d) $\neg(P \vee Q)$.
 - (e) $(Q \wedge P) \vee \neg P$.