NSC 562
DISCRETE MATHEMATICS
FALL 2017

Homework 4

The following exercises are from Daniel Velleman's *How to Prove It*.

(1) (§3.1 # 6) Suppose $a, b \in \mathbb{R}$. Prove that if $0 < a < b$ then $1/b < 1/a$.

(2) (§3.1 # 8) Suppose $A \setminus B \subseteq C \cap D$ and $x \in A$. Prove that if $x \notin D$ then $x \in B$.

(3) (§3.1 # 13) Suppose $x, y \in \mathbb{R}$. Prove that if $x^2 + y = -3$ and $2x - y = 2$ then $x = -1$.

(4) (§3.1 # 16) Consider the following incorrect theorem:

Incorrect Theorem. *Suppose that $x, y \in \mathbb{R}$ and $x \neq 3$. If $x^2 y = 9y$ then $y = 0$.*

(a) What's wrong with the following proof of the theorem?

Proof. Suppose that $x^2 y = 9y$. Then $(x^2 - 9)y = 0$. Since $x \neq 3$, $x^2 \neq 9$, so $x^2 - 9 \neq 0$. Therefore we can divide both sides of the equation $(x^2 - 9)y = 0$ by $x^2 - 9$, which leads to the conclusion that $y = 0$. Thus, if $x^2 y = 9y$ then $y = 0$. □

(b) Show that the theorem is incorrect by finding a counterexample.

(5) (§3.2 # 2) This problem could be solved by using truth tables, but don't do it that way. Instead, use the methods for writing proofs discussed so far in the chapter. (See Example 3.2.4.)

(a) Suppose $P \to Q$ and $R \to \neg Q$ are both true. Prove that $P \to \neg R$ is true.

(b) Suppose that P is true. Prove that $Q \to \neg(Q \to \neg P)$ is true.

(6) (§3.2 #4) Suppose that $A \setminus B$ is disjoint from C and $x \in A$. Prove that if $x \in C$ then $x \in B$.

(7) (§3.2 # 9) Suppose that x and y are real numbers. Prove that if $x^2 y = 2x + y$, then if $y \neq 0$ then $x \neq 0$.

(8) (§3.2 #12) Consider the following incorrect theorem:

Incorrect Theorem. *Suppose that $A \subseteq C$, $B \subseteq C$, and $x \in A$. Then $x \in B$.*

(a) What's wrong with the following proof of the theorem?

Proof. Suppose that $x \notin B$. Since $x \in A$ and $A \subseteq C$, $x \in C$. Since $x \notin B$ and $B \subseteq C$, $x \notin C$. But now we have proven both $x \in C$ and $x \notin C$, so we have reached a contradiction. Therefore $x \in B$. □

(b) Show that this theorem is incorrect by finding a counterexample.

(9) (§3.3#18) In this problem all variables range over $\mathbb{Z}$, the set of all integers.

(a) Prove that if $a|b$ and $a|c$, then $a|(b + c)$.

(b) Prove that if $ac|bc$ and $c \neq 0$, then $a|b$.