

NSC 562
DISCRETE MATHEMATICS
FALL 2017

HOMEWORK 1

- (1) Consider the following variation of the Tower of Hanoi: you have as many pegs as you desire, or “infinitely many” pegs, if you like. Again the objective is to transfer the entire tower (consisting of n decreasing disks), moving only one disk at a time and never moving a larger disk on top of a smaller disk.

Let R_n be the minimum number of moves it takes to perform this task, starting with n -many disks.

- (a) Draw a picture showing the moves required for $n = 0$, $n = 1$, $n = 2$, and $n = 3$.
(b) Explain in words how this recursive formula describes the method for moving the tower of n -many disks:

$$R_0 = 0$$

$$R_1 = 1$$

$$R_n = R_{n-1} + 2 \text{ for } n > 1.$$

- (c) Prove by induction that the following is the closed form solution: $R_n = 2n - 1$, for $n > 0$.

- (2) Warmup Exercise 1 from *Concrete Mathematics*.
(3) Warmup Exercise 5 from *Concrete Mathematics*.
(4) Warmup Exercise 7 from *Concrete Mathematics*.
(5) Homework Exercise 14 from *Concrete Mathematics*.