NSC 562
DISCRETE MATHEMATICS
FALL 2017

HOMEWORK 3

The following exercises are from Daniel Velleman's *How to Prove It*.

(1) (§1.5 # 2) Analyze the logical forms of the following statements:
   (a) Mary will sell her house only if she can get a good price and find a nice apartment.
   (b) Having both a good credit history and an adequate down payment is a necessary condition for getting a mortgage.
   (c) John will kill himself, unless someone stops him.
   (d) If x is divisible by either 4 or 6, then it isn't prime.

(2) (§1.5 # 3) Analyze the logical form of the following statement:
   (a) If it is raining, then it is windy and the sun is not shining.

   Now analyze the following statements. Also, for each statement determine whether the statement is equivalent to either statement (a) or its converse.
   (b) It is windy and not sunny only if it is raining.
   (c) Rain is a sufficient condition for wind with no sunshine.
   (d) Rain is a necessary condition for wind with no sunshine.
   (e) It's not raining, if either the sun is shining or it's not windy.
   (f) Wind is a necessary condition for it to be rainy, and so is a lack of sunshine.
   (g) Either it is windy only if it is raining, or it is not sunny only if it is raining.

(3) (§1.5 #9) Find a formula involving only the connectives $\neg$ and $\rightarrow$ that is equivalent to $P \leftrightarrow Q$.

(4) (§1.3 #4) Write definitions using elementhood tests for the following sets:
   (a) $\{1, 4, 9, 16, 25, 36, 49, \dots\}$.
   (b) $\{1, 2, 4, 8, 16, 32, 64, \dots\}$.
   (c) $\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$.

(5) (§1.4 # 2) Let $A = \{\text{United States, Germany, China, Australia}\}$, $B = \{\text{Germany, France, India, Brazil}\}$, and $C = \{x \mid x \text{ is a country in Europe}\}$. List the elements of the following sets. Are any of the sets below disjoint from any of the others? Are any of the sets below subsets of any others?
   (a) $A \cup B$.
   (b) $(A \cap B) \setminus C$.
   (c) $(B \cap C) \setminus A$.

(6) (§1.4 # 15) Fill in the blanks to make true identities:
   (a) $(A \triangle B) \cap C = (C \setminus A) \triangle$_____.
   (b) $C \setminus (A \triangle B) = (A \cap C) \triangle$_____.
   (c) $(B \setminus A) \triangle C = (A \triangle C) \triangle$_____.