test 2 :
numerical methods and
the damped oscillator

differential equations course, Marlboro College, April 2009, with Jim Mahoney

As always, the point here is to convince me that you understand the material. Clarity and approach count. Explain what's going on, don't just quote a result.

You may use sources (textbook, web, ...) but if so you **must** cite them explicitly, making clear what is your work and what isn't. That includes my class notes : if you're adopting my code (which is OK), say so. You are **not** allowed to work with other people or ask them for help: everything here should be an individual effort. You may use any numerical tools (*Mathematica*, python, excel, ...) you wish.

If you have questions about the test itself, find me and ask. If a problem is ill stated, make whatever assumptions you need to fix things, be clear about the issues, and do the problem in a way that makes sense.

deadlines : out Tues April 13 2009; due Tues April 20 by 8:30am

R C circuit

A resistor and capacitor in series are described by

$$R \frac{dI(t)}{dt} + \frac{I(t)}{C} = 0$$

where the variables, values, and their units (in the MKS system) are

t	time		
$I(t)$	current	with $I(0) = 2$ mA	$(1 \text{ mA} = 10^{-3} \text{ amp})$
R	resistance	$= 0.3\ \text{k}\Omega$	$(1\ \text{k}\Omega = 10^{3} \text{ ohm})$
C	capacitance	$= 4\ \mu\text{F}$	$(1\ \mu\text{F} = 10^{-6} \text{ farad})$

Don't let the units confuse you; the conversions are just :

$$1 \text{ sec} = \sqrt{(1 \text{ farad})(1 \text{ henry})} = (1 \text{ ohm})(1 \text{ farad}) = (1 \text{ henry})/(1 \text{ ohm})$$

In other words, these are standard MKS units; they multiply out to give seconds.

- **1. Use a Runge-Kutta numerical approach to find the time (to at least 3 digits accuracy) when the current drops to 1/100 of it's original value.**
- **2. Graph the solution.**

R L C Circuit

A resistor, an inductor, and a capacitor in series are described by

$$L \frac{d^2 I(t)}{dt^2} + R \frac{dI(t)}{dt} + \frac{I(t)}{C} = 0$$

where the variables, values, and their units (in the MKS system) are

t	time		
$I(t)$	current	with initial values $I = 2$ mA and $\frac{dI}{dt} = 0$ at $t = 0$.	
R	resistance	$= 0.3$ kΩ	$\left(1 \text{ k}\Omega = 10^3 \text{ ohm}\right)$
C	capacitance	$= 4\ \mu$F	$\left(1\ \mu\text{F} = 10^{-6} \text{ farad}\right)$
L	inductance	$= 1440$ mH	$\left(1 \text{ mH} = 10^{-3} \text{ henry}\right)$

- **3. Convert the equation to a system of first order equations.**
- **4. Use a Heun numerical approach for a system of equations to find the time (to at least 3 digits accuracy) when the current drops to 1/100 of it's original value.**
- **5. Graph it.**

Analysis

- **6. Describe the relation between the number of steps and the accuracy of your solution in these two cases. Which one took more time steps? Is that due to the numerical algorithm, or the equation itself? How do you know? Discuss.**