Feb 17 homework

Jim M, Feb 12 2009

1. Euler's Method

Solve this initial value problem using Euler's method :

$$\frac{dy}{dt} = \frac{3t^2}{3y^2-4} \text{ with } y = 0 \text{ at } t = 1$$

(a) Use a step size of $\Delta t = 0.1$, and solve up to at least $t = 1.8$.

(b) Use a step size of $\Delta t = 0.05$.

(c) Compare the two, and discuss.

(d) Sketch the direction field (feel free to use the code from the Mathematica notebook I showed in class).

(e) Mathematica's numerical differential equation solver is NDSolve. Read its documentation, try it out, and compare with your solution.

(f) Solve this exactly, and compare that, too.

(g) Suppose that the constants 3, 3, and 4 are unknown fixed values A, B, and C. If you still want to do this numerically, do you need to do a different solution for many values of all three? (Hint: suppose y is distance, and t is time. What units are $\{A, B, C\}$? How many can you get rid of by adjusting the distance and time unit?)

2. The Logistic Map

This problem asks you to investigate the logistic map. Use any software package you like to do so, including the Mathematica file I used in class or some code of your own, but do say which one you used.

(a) Confirm numerically for several values of r and $x_0$the the long term value does not depend on the initial value.

(b) Examine the period doubling transition near $r = 3$ by looking closely at (say) $r = \{2.9,\ 2.95,\ 2.99,\ 3.01,\ 3.05,\ 3.1\}$. In each case estimate the number of iterations required to reach a steady-state solution. How close is "close enough?"

(c) Define $r_k$to be the value of r where the period changes from $2^{k-1}$to 2^k. For example, the first period doubling happens at r_1, and $\{r_1, r_2, r_3\} \sim \{3.0, 3.449, 3, 544\}$. (Confirm this). It turns out that $(r_n - r_{n-1})/(r_{n+1} - r_n)$, which is the fractional distance to the next period doubling in terms of the last one, approaches a constant as $n \to \infty$, called "the Feigenbaum number", which is approximately 4.6692. This number is interesting because it turns out to be universal, showing up in all period-doubling approaches to chaos, irregardless of the details of the system. Find the value of this ratio for the smallest period doublings, and see how how far off that is as a percentage from Feigenbaum's number.