Integration by Parts

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for some constant C.

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We cannot use integration by substitution to do the following calculation:

$$\int xe^x\,dx. \tag{2}$$

We have to use a different method.

Differentiation Recap

Recall the product rule of differentiation:
Let $f(x)$ and $g(x)$ be two differentiable functions. Then

$$(f(x) \cdot (g(x))' = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

By the Fundamental Theorem of Calculus:

$$\begin{aligned} \int (f(x) \cdot (g(x))' \, dx &= \int f(x) \cdot g'(x) dx + \int f'(x) \cdot g(x) dx \\ (f(x) \cdot (g(x)) &= \int f(x) \cdot g'(x) dx + \int f'(x) \cdot g(x) dx, \end{aligned}$$

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Hence

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx. \tag{3}$$

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Formulas

Hence

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx. \qquad (3)$$

Let $u = f(x)$ and $dv = g'(x)dx$.
Then $du = f'(x)dx$ and $v = g(x)$.
So Equation 3 gives

$$\int u\, dv = uv - \int v\, du.$$

Mnemonic: Ultra-Violet Voodoo.

Dont forget to include the term dx with the terms u and dv!
Indefinite version:

$$\int_a^b u\, dv = uv|_a^b - \int_a^b v\, du$$

Example

Recall Example 2,

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So

- $du = 1\,dx$ and $v = e^x$.

Thus

$$\begin{aligned}\int u\,dv &= uv - \int v\,du \\ &= xe^x - \int e^x \cdot 1\,dx \\ &= xe^x - e^x + C,\end{aligned}$$

for some constant C.

Integration by Parts: Advice.

- How do we choose what to differentiate and what to integrate?
  - According to *LIPET!!*

Preference for Differentiation: Best top, worst bottom.

L is for Logarithm;

I is for Inverse Trig Functions;

P is for Polynomials;

E is for Exponential Functions;

T is for Trig Functions.

Note: It is better to integrate by substitution than it is to use integration by parts.

Examples & Drills: Link