Complex Variables

- Algebra
- Complex conjugate
- Distance between two complex numbers
- Triangle inequality

Complex Variables

- Sets of numbers:
- Let C be the set of all complex numbers
- Let N be the set of all natural numbers.
- Let Q be the set of all rational numbers.
- Let R be the set of all real numbers.
- Let Z be the set of all integers.

Complex Variables

- Closure:
  - Any 2 numbers from a set together with some operation remain in the set.
  - What sets are closed under what operations?
    - Addition; Subtraction;
    - Multiplication; Division (excluding 0).

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  - Algebraically closed: Roots of a polynomial in one variable with integer (or equivalently rational) coefficients.

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Complex Variables

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  - Alternate view: Define i such that $i^2 = -1$

We then define complex numbers as follows:

$$\{a + bi : \ a, b \in \mathbb{R}\}$$

Complex Variables

- USE ALGEBRA!!
- Let a, b, c and d be real numbers.
  - $(a+bi)+(c+di)=a+c+(b+d)i$
  - $(a+bi)-(c+di)=a-c+(b-d)i$
- For multiplication:
  - $(a+bi)*(c+di)=ac+adi+bci+bdi^2$

    $=(ac-bd)+(ad+bc)i$ since $i^2=-1$.

Complex Variables

- Example:
- Let z_1=1+2i and z_2=-3-5i.
  - z_1+z_2=-2-3i
  - z_1-z_2=4+7i

$$\begin{aligned} z_1 \cdot z_2 &= (1+2i)\cdot(-3-5i) \\ &= -3-5i-6-10i^2 \\ &= 7-11i \end{aligned}$$

Complex Variables

- To try:
  - Let $z_1 = 2 + 5i, z_2 = 6 - 8i.$

Work out the following:

$$z_1 + z_2; \quad z_1 - z_2 \quad z_1 \cdot z_2$$

Complex Variables

- Definition

Let z=a+bi be a complex number. It's complex conjugate is the number $\overline{z} = a - bi$

Note:
$$\begin{aligned} z \cdot \overline{z} &= (a+bi)\cdot(a-bi) \\ &= a^2 - abi + abi - b^2 i^2 \\ &= a^2 + b^2 \end{aligned}$$

Since i^2=-1.

Complex Variables

- Example:
  - 1+i is the complex conjugate of 1-i
  - -3-2i is the comlpex conjugate of -3+2i.

Complex Variables

- Note that we have multiplied two complex numbers to get a real number!
- We use complex conjugates in the division of complex numbers. The aim is to get rid of the terms involving i in the denominator

$$\frac{a+bi}{c+di}$$

Complex Variables

- General Rule: Using i^2=-1.

$$\begin{aligned}\frac{(a+bi)}{(c+di)} &= \frac{(a+bi)}{(c+di)}\frac{(c-di)}{(c-di)} \\ &= \frac{ac-adi+bci-bdi^2}{c^2+d^2} \\ &= \frac{(ac-bd)+(bc-ad)i}{c^2+d^2}\end{aligned}$$

Complex Variables

- Example:

$$\begin{aligned}\frac{(1+2i)}{(-3-5i)} &= \frac{(1+2i)}{(-3-5i)}\frac{(-3+5i)}{(-3+5i)}\\ &= \frac{-3+5i-6i+10i^2}{9+25}\\ &= \frac{-13-i}{34}\end{aligned}$$

Complex Variables

- To Try:
  - What is (2+5i)/(6-8i)?
- Pauls Online Notes on Conjugate and Modulus