

Topics in Algebra, Trigonometry and Pre-Calculus

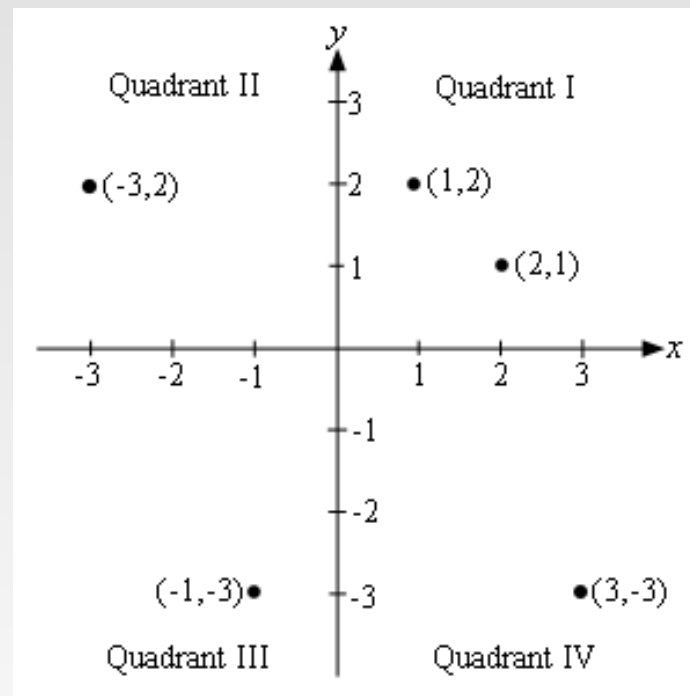
- The Cartesian co-ordinate system

Topics in Algebra, Trigonometry and Pre-Calculus

- The Cartesian co-ordinate system
 - Co-ordinates;
 - Distance between points;
 - Mid-point of line segment;

The Cartesian co-ordinate system

- Standard x and y-axis



The Cartesian co-ordinate system

- Quadrants:
 - They go anti-clockwise starting from the quadrant where both x and y are positive.

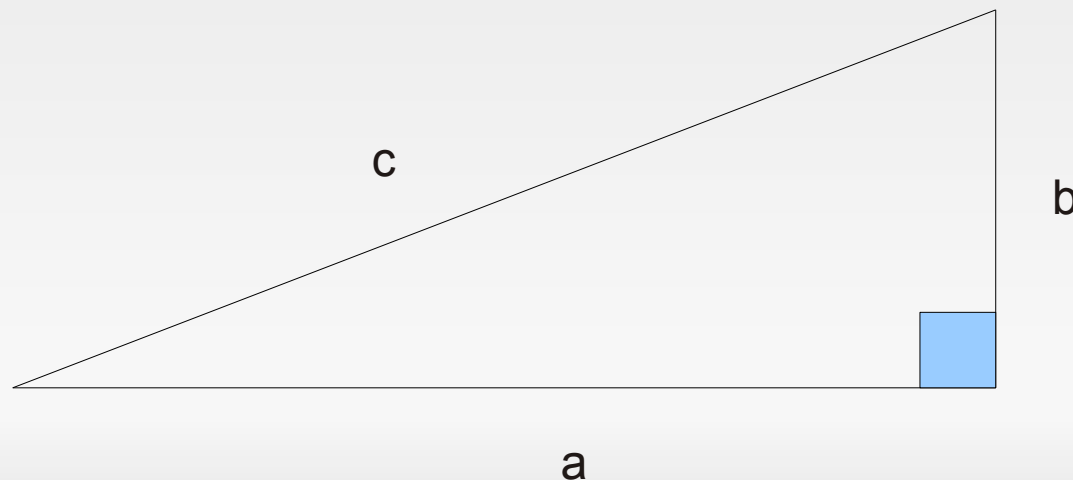
The Cartesian co-ordinate system

- Quadrants:
 - They go anti-clockwise starting from the quadrant where both x and y are positive.
 - They have an important role to play with Trigonometric functions. Discuss in later unit.

The Cartesian co-ordinate system

- Distance between points:
 - Pythagoras' Theorem
 - Let a , b , c be the lengths of the sides of a right-angled triangle such that c is the longest length. Then:

$$c^2 = a^2 + b^2$$

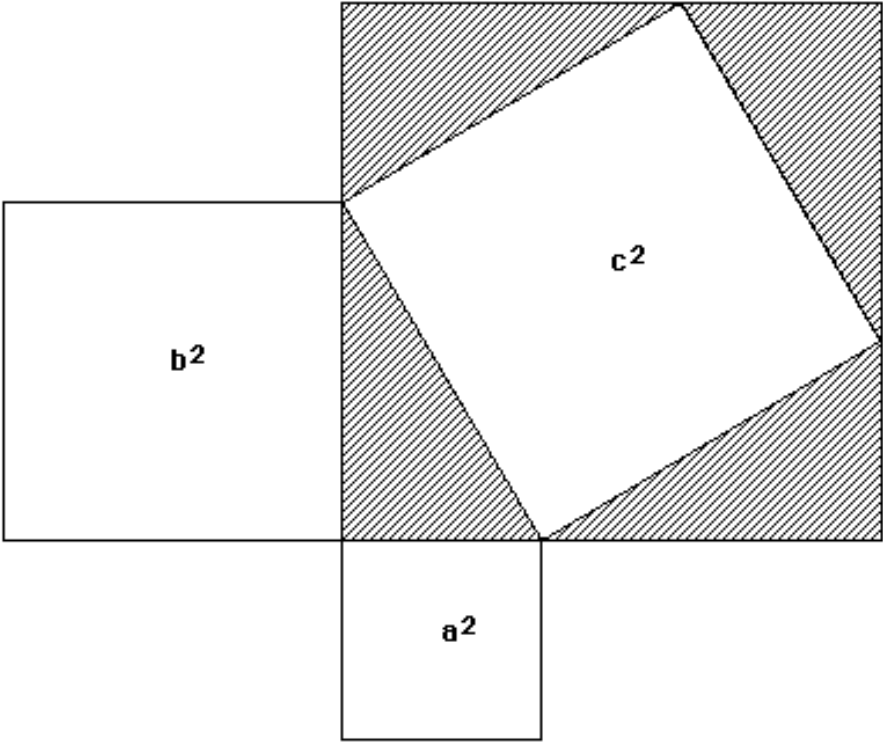


The Cartesian co-ordinate system

- [Link to picture](#)

PYTHAGORAS'S THEOREM

In a right angled triangle the area of the square on the hypotenuse is the sum of the areas of the squares on the other two sides.



HERE IS A PROOF:

Fit copies of the triangle around c^2 .

The area of the big square is area $(a+b)^2$

The triangle's area is $ab/2$.

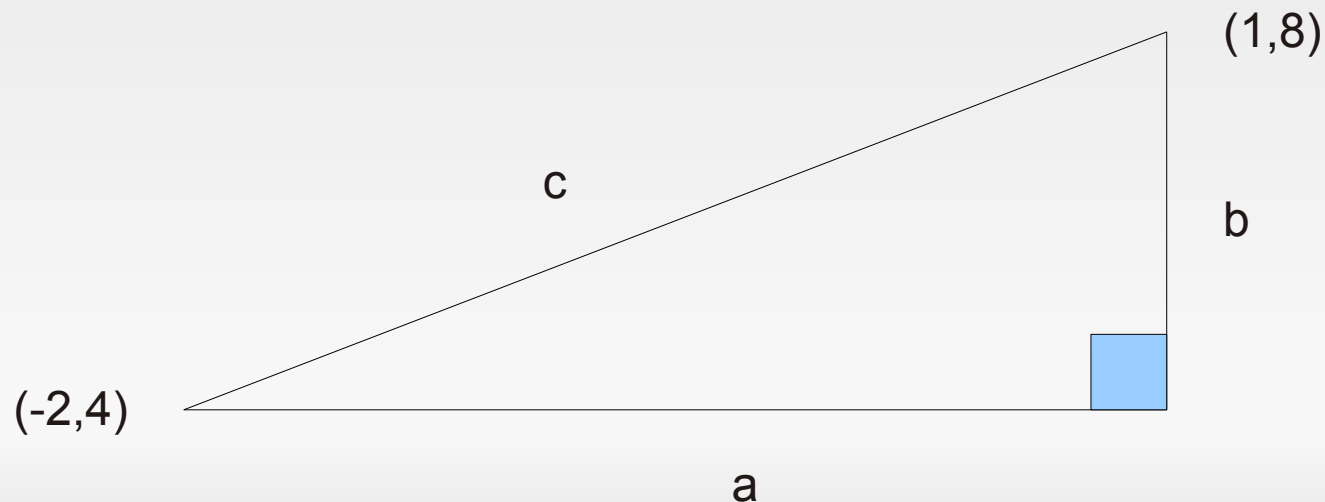
Hence $(a+b)^2 = c^2 + 4(ab/2)$.

So $a^2 + 2ab + b^2 = c^2 + 2ab$

and thus $a^2 + b^2 = c^2$.

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- YouTube: Easy [proof!](#)
- Example:
 - What is the distance between the points $(-2,4)$ and $(1,8)$?



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- The distance c is calculated as follows using Pythagoras' theorem:

$$c^2 = ((-2) - 1)^2 + (4 - 8)^2$$

$$c^2 = 25$$

$$c = 5$$

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- Question:
 - What are the distances between each of the following pairs of points
 - a) $(4,3)$ and $(-1,9)$?
 - b) $(-11,-11)$ and $(-3, -5)$?
 - c) $(6,5)$ and $(15,17)$?

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- Answers:
 - a) 13.
 - b) 10.
 - c) 15.

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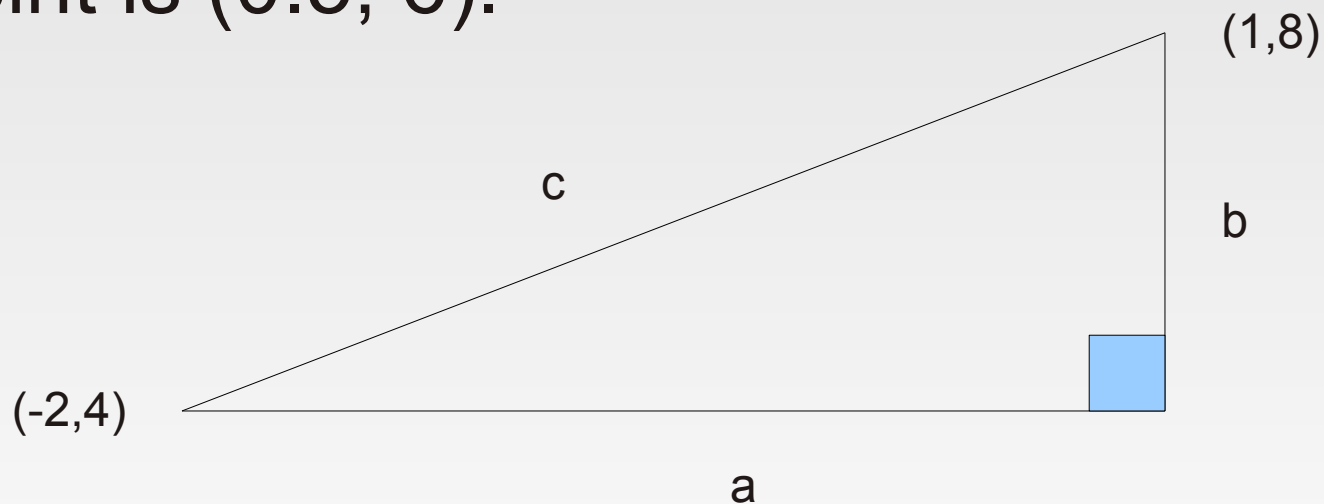
- The mid-point of two points:

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- The mid-point of two points:
 - Starting at one of the points, the mid-point is clearly halfway up (or down) and halfway along.

- Example: What point is halfway between $(1,8)$ and $(-2,4)$?

- Answer: We compute the triangle has sides $a=3$, $b=4$ and $c=5$.
- We find that $a/3=1.5$ and $b/2=2$, and so the midpoint is $(0.5, 6)$.



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- Question:
 - What are the mid-points between each of the following pairs of points
 - a) $(4,3)$ and $(-1,9)$?
 - b) $(-11,-11)$ and $(-3,-5)$?
 - c) $(6,5)$ and $(15,17)$?

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- Answers:
 - a) $(1.5, -3)$
 - b) $(-7, -8)$
 - c) $(10.5, 11)$