Complex Numbers and Inner Products : Fourier Transform warm-up

Jim Mahoney, Marlboro College, Feb 2011

1. $7\, e^{0.3\, i} = x + i\, y$. Find x, y .

2. $7 + 0.3\, i = R\, e^{i\,\theta}$. Find R, θ .

3. $(2 + i)\,(3 - i) = ?$

4. Convert the complex numbers in the last problem to polar form. Show that in the multiplication, the angles add.

5. $(2 + 3\, i)^{(1/3)} = ?$

6. $\cos(1 + i) = ?$

7. $i^i = ?$

Define three vectors as follows :

$$\hat{\boldsymbol{e}}_1 = \left(\frac{1}{2},\ \frac{1}{2},\ \frac{1}{\sqrt{2}}\right)$$

$$\hat{\boldsymbol{e}}_2 = \left(\frac{1}{2},\ \frac{1}{2},\ -\frac{1}{\sqrt{2}}\right)$$

$$\hat{\boldsymbol{e}}_3 = \left(\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}},\ 0\right)$$

8. Show that these three vectors form an ortho-normal basis. In other words, that each has unit length, and that they're all perpendicular. (Hint: all of this is done with the inner "dot" product.) Sketch a picture of where they look like in the traditional (x, y, z) space.

9. Let

$$\vec{\boldsymbol{a}} = \hat{\boldsymbol{i}} + 2\,\hat{\boldsymbol{j}} + 3\,\hat{\boldsymbol{k}} = (1,\ 2,\ 3)$$

Find the coordinates of $\vec{\mathbf{a}}$ in the $\hat{\boldsymbol{e}}$ basis.
That is, find three numbers a_1, a_2, a_3 such that

$$\vec{\boldsymbol{a}} = a_1\,\hat{\boldsymbol{e}}_1 + a_2\,\hat{\boldsymbol{e}}_2 + a_3\,\hat{\boldsymbol{e}}_3 = \sum a_i\,\hat{\boldsymbol{e}}_i$$

Are we having fun yet?