

## Puzzle Set 2: Mathematical History

This set of puzzles features a host of problems, mostly 100+ years old, that have inspired “serious” mathematical work. Sometimes the puzzles have essentially founded whole branches of mathematics.

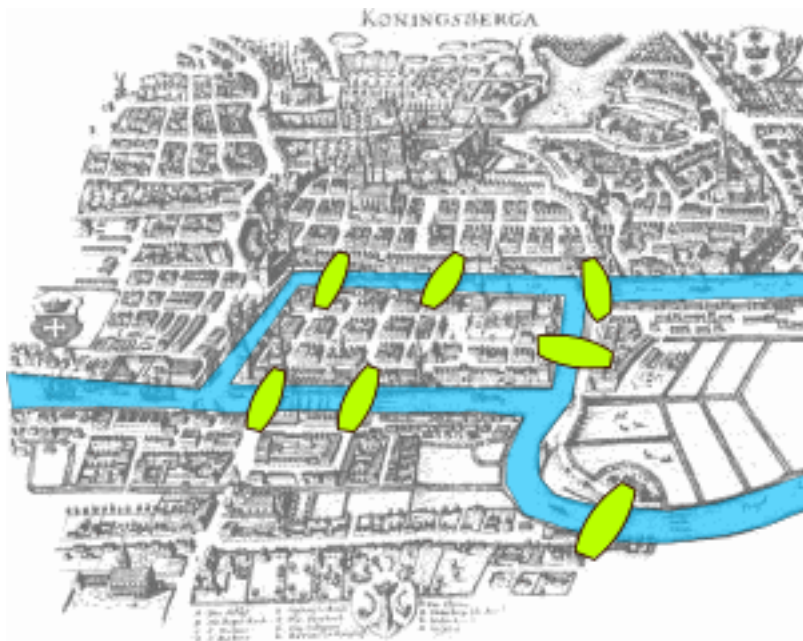
1. We’ve seen (and solved) this one already:

(On the Ramsgate Sands, H. E. Dudeney, 1925). Thirteen youngsters were seen dancing in a ring on the Ramsgate Sands. Apparently they were playing “Round the Mulberry Bush”. The puzzle is this. How many rings may they form without any child ever taking twice the hand of any other child—right hand or left? That is, no child may ever have a second time the same neighbour.

Moreover, we solved it for all odd prime numbers (not just 13). The challenge now: can you solve it for nine children? 15? All odd numbers?

2. The Cocktail Party Problem. Show that at any cocktail party at which there are six people in attendance there are either three mutual acquaintances or three mutual strangers (or both). Is this true at parties with five people?

3. Consider the following picture of Königsberg:



Can you find a route for a walk through the city that crosses each bridge exactly once and ends where you started? More generally, can you come up with a method of quickly determining a route (or a proof that no route exists) that works for any map?

4. Choose a positive whole number. If it is even then halve it. If it is odd then multiply it by 3 and add 1. You now have a new number. Repeat the process (halving if even, multiplying by 3 and adding 1 if odd) to get a third number, and then a fourth, fifth, sixth, and so on. What happens? Does it happen with *every* starting number?
  
5. Six regiments each send a colonel, a lieutenant-colonel, a major, a captain, a lieutenant, and a sub-lieutenant to a parade. How may the 36 soldiers be arranged into a six by six formation such that no regiment is repeated in any row or column and no rank is repeated in any row or column?  
  
Easier place to start: what if there are three regiments with three ranks and you need a  $3 \times 3$  grid of the nine soldiers? Or four regiments and ranks and you want a  $4 \times 4$  grid? Etc.
  
6. Are there infinitely many primes? Can you prove it?
  
7. Suppose there are three cottages on a plane (or sphere) and each needs to be connected to the gas, water, and electric companies. Using a third dimension or sending any of the connections through another company or cottage are disallowed. Is there a way to make all nine connections without any of the lines crossing each other?
  
8. (Kirkman's Schoolgirl Problem, 1847) Fifteen young ladies in a school walk out three abreast for seven days in succession: it is required to arrange them daily so that no two shall walk twice abreast.